

CHAPTER VIII- RISK AND UNCERTAINTY

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I - INTRODUCTION

There is a classical distinction between **risk** and **uncertainty** developed by Knight in 1921.

Risk is defined “as a *measurable uncertainty*: risk goes with probabilities while uncertainty goes with *unknown probabilities*”.

II – RISK

Let’s start from M. Dorfman’s example, given in 1972¹. It’s a situation where you have to make a decision choice. The decision choice involves the extent to which to fill the reservoir. If the flood does not come, the outcome will be greater if one spills more. If there is a flood, net benefits are greater if one spill less. If one spills all, there is no flood protection left and the net benefits are the same wither there is a flood or not.

The probability of the flood occurring is judged to be 0.4, which lean that 1 minus 0,4 (0,6) is the no flood probability.

2.1 - Expected value versus utility

The expected value

The expected value is the sum of possible outcomes weighted by their probabilities. It has the meaning of an average outcome, that is the value one would observe as the outcome on the average, if the project were to be carried out a large number of times.

¹ Dorfman’s M. (1972)

Table 1. Expected value of outcome

Decision	Flood	No flood	Expected value of outcomes
Spill 1/3	130	400	$292=0,4*130 + 0,6*400$
Spill 2/3	140	260	212
Spill all	80	80	80
Probabilities	0,4	0,6	

Considering the option to spill one third in the reservoir.

Example.

The EV is $(0,4)\$130+0,6(\$400)=\$292$.

In diagram EV appears on the horizontal income axis and is denoted by \bar{Y} . If the probability of the flood occurring were 1; the EV would be \$130.

If the flood had a probability, the EV would be \$400. As the flood is not certain, \bar{y} is located between \$130 and \$400.

The \bar{y} of \$292 is nearer to \$400 because the relative probability is greater than the flood will not occur. Using EV is one way of deciding among uncertainty outcome. The decision rule would be to choose the option with the highest EV. The EV is \$292 for the one third spill. This is highest than either of the other two options (\$212, \$80). Thus the one third spill option would be chosen if the objective were to maximise the EV.

Expected utility

An alternative way of considering outcomes **is in terms of utility** values of the dollar figures.

Next table shows the corresponding utility values that Dorfman assigned to each dollar outcome.

The **expected utility** EU is defined in an analogous way to the EV.

It is the sum of possible utility outcome weighted by their probabilities.

The EU appears on the vertical axis of the next diagram and is denoted by U. Thus for the one spill option the EU is: $(0,4)0,30+(0,6)1,15=0,81$

Using expected utility is an alternative way of deciding among alternative.

As the one third spill option has an expected utility greater than the other two (0,81 is larger than 0,61 et -0,23); this would be the most preferred option when one tries to maximise the EU.

Table 2. Expected utility

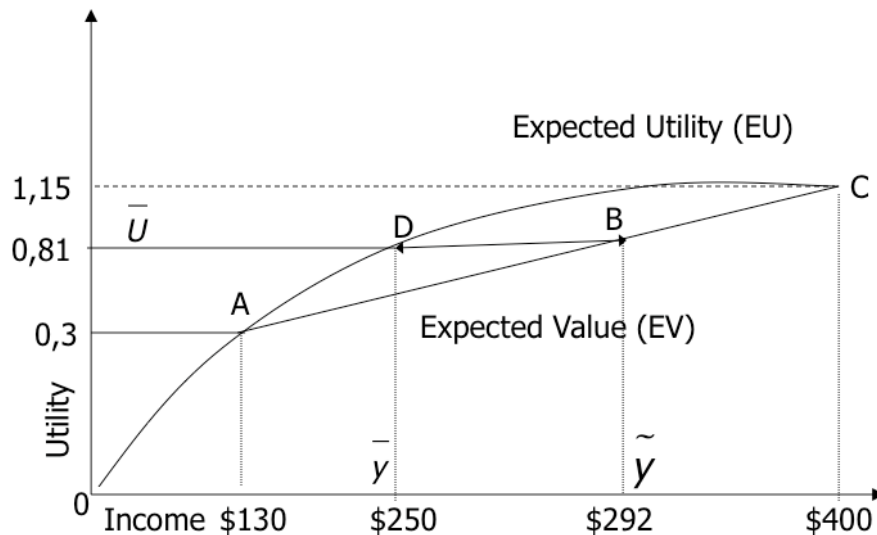
Decision	Flood	No flood	Expected value of utility
Spill 1/3	0,3	1,15	0,81
Spill 2/3	0,37	0,90	0,68
Spill all	0,23	-0,23	-0,23
Probabilities	0,4	0,6	

If an outcome is to be socially optima, the aim must be to maximize utility not to maximize income.

There would be no difference in the two criteria if there were a simple, proportional relationship between income and utility.

The straight line in the diagram depict such a linear relationship. But usually, one assumes that there is a *diminishing marginal utility of income (that is, the more income one has, the less the additional satisfaction)*.

The utility curve ADC in the next diagram is drawn with a diminishing marginal utility of income property (it is concave from above and convex from below). All other concepts that are to follow help clarify the essential difference between the linear and the non-linear case



Graph 1. Risk aversion

The relation between utility and income is drawn as the curve ADC. This shows the risk aversion. Risk neutrality is depicted by the straight line ABC. The cost of the risk is the horizontal distance DB between these two relations at the expected utility level 0,81, being the difference between the EV level of income \$292 and a certainty equivalent income \$250.

2.2 – The certain equivalent and the cost of risk

We call a **certain equivalent income** the level of a sure income that gives the same level of utility as a lottery with the same expected utility.

One can see the reservoir project as a lottery.

With the first option we “win” if the flood doesn’t occur and the utility value 1,15 is obtained. The lottery is lost when the flood goes and the utility value 0,3 results. The certain equivalent should have to convert the set of uncertain outcome to a figure known with certainty.

The line ABC shows all the combination of 0,3 utility and 130 outcome that corresponds to each probability values of the lottery.

When the probability is 0,4 (which means that the value 1,15 occurs with a probability 0,6 and the value 0,3 occurs with a probability 0,4) we obtain the point B (this was how the EV value of 0,81 was calculated).

Point A would be when the probability of flood occurrence was 1. C would be when the probability was 0.

On the graphic, we have a line ADC that shows the utility level for any level of certain income. This curve ADC is above ABC because individuals prefer to have 292 with certainty than a lottery with an expected value of 292.

They are indifferent between the expected value of income of a 292 lottery and the utility of certain income of 250. 292-250 is the cost of the risk. DB describes the risk adverse-ship.

Because curve ADB is above line ABC, the certain equivalent of point B is valued less than the expected value. To obtain the same level of satisfaction as B (EV of 0,81) one must move to D or the certain utility curve. Thus 250 is the certain equivalent of expected value of 292.

The cost of risk quantifies the difference between the two relations ADC and ABC. **K is defined as the difference between a project expected value and its certainty equivalent** income:

$$k = \bar{Y} - Y = K = 250 - 292 = 42$$

K is the cost of risk. People are ready to give up 42 dollars if they could obtain 250 for certain rather to face the risky project with an expected value of 292.

If K is equal to 0 they are risk neutral.

If K is positive they are risk adverse.

If K is positive they are risk lovers.

III - DEALING WITH RISK

Is the fact there is uncertainty a good reason to modify the discount rate?

The net present value of a project is minus the cost plus benefit. If you deal with more than one period you have to discount the benefit:

$$NPV = -CO + \frac{B}{(1+i+\rho)} \quad (1)$$

Where i is the discount rate and ρ is the risk premium.

If the project costs 100 today and make 100 tomorrow the project is not good. Usually we assume that the long time interest rate is the second best alternative so i (the discount rate) is assumed as equal to the long time interest rate.

Do we think that we have to add a risk premium to the function (ρ)?

3.1 - How to assess a cost of risk

The cost of risk K is the difference between the expected value of benefit B and the certainty equivalent B^* . Thus we can obtain the present certainty equivalent value:

$$PCEV = -CO + \frac{(B-K)}{(1+i)} \quad (12)$$

Instead of valuing K one can obtain a numerical equivalent for this function by valuing the risk premium ρ .

If $B=1,2$ $Co=1$ $K=0,1$ and $i=0,1$, then the PCEV would equal to 340.

If $B=1,2$ $Co=1$ $i=0,1$ we can provide that $\rho=0,1$ (by the NPV function).

3.2 – The Arrow-Lind theorem

We know that there is uncertainty and that this is a good reason to add a risk premium to discount future benefit to public policy evaluation.

The theory says that there is a good reason not to put the risk premium.

The Arrow-Lind theorem says that we should not use a risk premium under two assumptions:

- The **return from the public project must be individually independent of national income**. The public project should not have correlation with the private sector. If there is a positive correlation, $\rho > 0$ is required. If there is a negative correlation $\rho < 0$ is required. Public projects are dedicated to the all society so there is a pooling mechanism for risk. We assume that the mix is risk neutral (risk lovers plus risk adverse). If the public project is correlated with a private sector then some people will be affected by the public project (and only this group). If there is no correlation, the theorem applies and you don't need a premium.
- The **returns of the public projects must be spread out over a large number of individuals**. The larger it is the better the pooling mechanism works.

The validity of the theorem depends on two assumptions.

- The first assumption is particularly hard to justify. Even if the production function is such that the project itself gives a return unrelated to income in its absence, the fact that the government taxes income in the absence of the project ensure some correlation. For, in order to finance the public project tax will have to be adjusted (The Foldes-Rees (1977) theorem says exactly this).
- The second assumption implies that the group variance will fall as the number increases. But when externalities and public good exists (the n on rival characteristic is present) as they do with most public investment, the risk per person is not reduced when the number of

individuals involved is increased (this argument is due to James 1975).

From the point of view of this text, the key criticism of the theorem involves **its neglected of distributional considerations**. Project should favour groups that would be poor in the absence of the project. A negative correlation is important a negative risk premium should be used to lower the discount rate.

3.3 – Adjusting the discount rate for risk

The first question to ask is whether any adjustment needs to be made to the social discount rate because of risk?

The answer is clear with an **individualistic framework**. For, if private individuals adjust for risk due to risk aversion social decision based on individual preference **must also adjust** for risks.

The conclusion would be otherwise if, when **aggregating**, individuals risks cancel out (strictly, disappear in the limit). But the two conditions necessary for this result (Arrow-Lind Theorem) **are unlike to exist**. The expected value of benefits needs to **be reduced by the cost of risk**, which implies to use a positive risk premium.

The next question is **whether public sector** should make the same cost of risk adjustment as the private sector?

The answer in general, the public sector should not make the same risk adjustment. We saw that what was important in the formulation of risk was the covariance, between a particular project and the state of the economy in the absence of the project. One should expect (for all reasons explained) that the public sector would undertake different projects from the private sector. Hence the covariance would be different and so would the risk adjustment.

Finally what do the previous section say about the common practise of adding risk premium to the risk discount rate in an ad hoc fashion?

Firstly, especially when the public sector has distributional objectives, there may be a negative covariance between public project and the economy in the absence of such projects. Here it is appropriate to reduce the discount rate rather than raise it.

Secondly there is precise way of determining just how large the adjustment to the discount rate should be (Zerbe and Dively, 1994). Not just any adjustment is appropriate.

Thirdly, precise adjustment can be made only in the context of a two periods model. The common practice adds a risk premium to the discount rate for each and every period. This can be correct only if uncertainty increase over time. In general, this is not a correct assumption.

4 – CONCLUSION

The common practice is to add a risk premium but you have to discuss how important this premium has to be. Most of the time bad analysts are saying the “there is a 10% risk than benefit occur in 90%”. They put a 10% premium and they kill the project. There is no reason to put % premium than there is equal x% risk.

The most common case of risk premium occurs when you are dealing with environment risks. The second common case is the one of health treatment (example of cancer).