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CHAPTER III– SOCIAL DISCOUNT RATE

I – INTRODUCTION

Given a choice, individuals would prefer to have a unit of benefits **today** rather than in **the future**.

The current resources can **build new resources** in the future.

Purpose: This chapter deals with the theoretical issues pertaining to the selection of an appropriate *real* social discount rate (SDR).

II – BASIS

When evaluating government policies or projects, analysts must decide on the appropriate weights to apply to policy impacts that occur in different years.

Given these weights, denoted by w_t, and estimates of the real annual net social benefits, NB_t, the estimated net present value (NPV) of a project is given by:

$$\text{NPV} = \sum_{t=0}^{n} w_t \text{NB}^t \qquad (1)$$

This formula is equal to the more commonly used:

$$NPV = \sum_{t=0}^{n} \frac{B_t}{(1+r)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+r)^t} \qquad (2)$$

when:

$$w_t = \frac{1}{(1+r)^t} \text{ and } i \geq 0. \text{ I} \qquad (3)$$

In this second equation we implicitly assume that the social discount weight declines at a constant rate. In this chapter will assume that in some situations it is not the case.

Selection of the appropriate social discount rate is equivalent to deciding on the appropriate *set of weights* to use in equation (1).

Sometimes the weights are referred to as *social discount factors*.

Discounting reflects the idea that a given amount of real resources in the future is worth less today than the same amount is worth now.

This is because:

1. Via investment, one can transform resources that are currently available into a greater amount in the future.

2. People prefer to consume a given amount of resources now rather than in the future.

Thus, it is generally accepted that the social discount weights decline over time; specifically, $0 < w_n \leq w_{n-1} \leq \ldots \leq w_1 \leq w_0 = 1$.

However, there is not so much agreement about the values of the weights. The key issue in this chapter is to decide on the weights.

Three unresolved issues are pertinent:

1. Whether market interest rates can be used.

2. Whether to include unborn future generations.

3. Whether society values a unit of investment the same as a unit of consumption.

Different assumptions about these issues lead to different discount rate methods, which, in turn, lead to different discount weights. There is considerable disagreement about the underlying assumptions and, therefore, about the most appropriate method. There is reasonable consensus over the discount weights appropriate for each method.

Does the choice of discount rate matter?

Table 1 – Annual net benefits and NPV
(Values for three alternative projects)

Year	Project 1	Project 2	Project 3
1	-80,000	-80,000	-80,000
2	25,000	80,000	0
3	25,000	10,000	0
4	25,000	10,000	0
5	25,000	10,000	140,000
NPV (i=2%)	37,838	35,762	46,802
NPV (i=10%)	14,770	21,544	6,929

Yes – choice of rate can affect policy choices. (Do the same calculation on a excel spreadsheet for i=12%).

Generally, low discount rates favour projects with the highest total benefits, while high SDRs rates favour projects where the benefits are front-end loaded.

III – THE THEORY BEHIND THE APPROPRIATE SOCIAL DISCOUNT RATE METHOD IN PERFECT MARKETS

To understand the theoretical foundation of discounting, we must recognize that it **is rooted in the preferences of individuals**.

Individuals tend to prefer to consume immediate benefits to ones occurring in the future.

Individuals also face an opportunity cost of forgone interest when they spend dollars today rather than invest them for future use.

These two considerations of importance to individual decisions -- the *marginal rate of time preference* and the *marginal rate of return on private investment* -- provide a basis for deciding how costs and benefits realized by society in the future should be discounted so that they are comparable to costs and benefits realized by society today.

1. An Individual's Marginal Rate of Time Preference (MTRP)

An individual's MRTP is the proportion of additional consumption that an individual requires in order to postpone (a small amount of) consumption for one year.

Equality of Discount Rates in Perfect Markets

In a perfectly competitive capital market, an individual's MRTP equals the market interest rate, as shown in Figure 1.

In this two-period model, an individual may consume her entire budget (T) in the first period, she may invest it all in the first period and consume T(1 + *i*) in the second period, or she may consume at any intermediate point, which is represented by the budget constraint in Figure 1 that has a slope of -(1 + *i*).

The absolute value of the slope of the constraint budget exceed 1 indicating that the consumer earns positive interest at rat r on the part of income saved in the year.

Consumption is maximised at the point where the indifference curve is tangent to the budget constraint, i.e. at point A.

At point A, the slope of the indifference curve is -(1+p), the marginal rate of substitution (MRS) is 1+p, and MRTP is p.

Consequently, $i = p$. Note that as current consumption increases, MRS and MRTP decrease.

Formally, The consumers maximize his utility U(.) subject to a budget constraint in which t denote the present value of total income over the two years and I the market interest rate.

$$\begin{cases} Max\ U(C_1, C_2) \\ s.c\ \ C_1 + \dfrac{C_2}{1+i} = t \end{cases} \tag{4}$$

The lagrangian is:

$$L = U(C_1, C_2) + \lambda(C_1 + \frac{C_2}{1+i} - t) = 0 \tag{5}$$

The first orders conditions ar :

$$\begin{cases} L_{C1} = \dfrac{\delta U}{\delta C_1} - \lambda = 0 \\ L_{C2} = \dfrac{\delta U}{\delta C_2} - \dfrac{\lambda}{1+i} = 0 \end{cases} \tag{6}$$

Consequently:

$$\frac{\delta U}{\delta C_1} \Big/ \frac{\delta U}{\delta C_2} = 1 + i \qquad (7)$$

Now, by definition, $MRS = \frac{\delta U}{\delta C_1} \Big/ \frac{\delta U}{\delta C_2} = 1 + p$, where p=marginal rate of time preference.

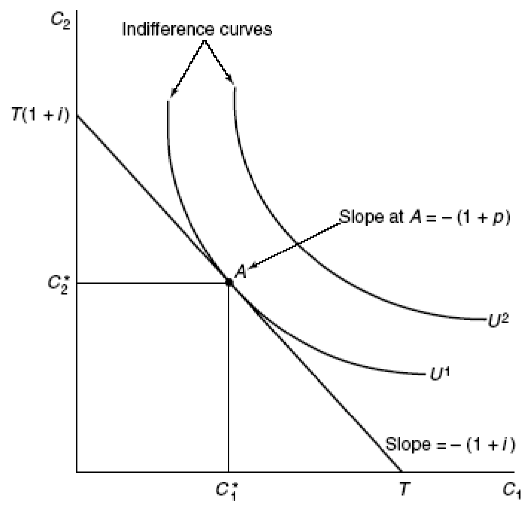
Hence, i=p at the optimum.

The result holds generally for people without extreme preferences over consumption in the two periods.

Formally the result holds as long as the problem hs an interior solution. A very impatient person has an MRTP>i at all points on his indifference curve, which leads to a corner solution on the C_1 axis.

Thus, every individual has an MRTP equal to the market interest rate. Because all consumers face the same market interest rate in an economy with a perfect capital market, all consumers have the same MRTP. Thus everyone is willing to trade current and future consumption at the same rate. Consequently, it is natural to interpret this rate which equal the interest rate, as the social discount rate.

Figure 1 – Equity of the MRTP and the interest rate



Rate of Return on Private Investment Equals the Market Rate Equals MRTP

We now present a more general, two-period model that pertains to a group of individuals in a hypothetical country and that incorporates production.

We assume that this country does not trade with other counties. Moreover, as previously, we initially ignore taxes, risks faced by private-sector investors and lenders, and transaction costs associated with making loans.

Consequently, anyone who wants a loan can borrow the desired amount at the market interest rate. In addition, we ignore market failures such as externalities and information asymmetry, which could cause private and social discount rates to diverge from one another.

The curve labelled CPF is a consumption possibility frontier that represents all the combination of current and future consumption that are feasible if the economy utilizes its resources efficiently. A major difference between this model and the previous is that this model incorporates production.

Suppose that at the beginning of the first period, the country has T units worth of resources, which can be allocated between current consumption and investment. The society could consume all T units in the current period, investment and, therefore have no future consumption s represented by point T. At the other extreme society could consume no units in the current period, invest all T units, and consume S units in the future period. Note that T>S, implying a positive rate of return on all resources invested in the current period.

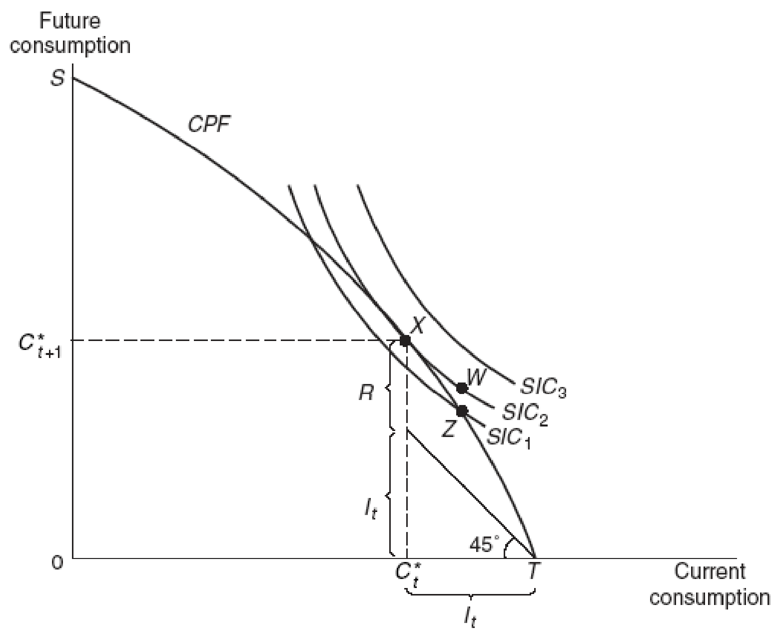
Point X represents a more realistic intermediate position where current consumption equals C_t^* and future consumption C_{t+1}^*. At this point society would relinquish $I_t = T - C_t^*$ units of potential current consumption for investment. But in the future consume $C_{t+1}^* = I_t + R$ units, where I_t the amount invested in the first period and R the return on investment. The partitioning of the future consumption into these two components is represented graphically by drawing a 45° line from point T to the line between X and C_t^*. The lower segment correspond to I_t and the higher segment to R.

The total investment I_t can be thought as a number of smaller investments that sum to I_t. The average rate of return on these investments, r, equal $R/I_t \; I_t$. This differs from the marginal return on a (Small) investment at X, R_x. To see this, note that when viewed from below, the CPF curve is concave. This implies that as the economy moves along the CPF curve from T toward S, and more resources are invested in the current period , returns are smaller. Consequently the marginal rate of return on investment at X is smaller that the average return between T and X, r.

By definition $-(1+r)$ equals the slope of the CPF at X, whereas the average slope of the CPF between T and X equals $-C_{t+1}/I_t = -(1+r)$. Because of the concave shape of the CPF, the slope at X is not as steep as the average slope between X and T, and therefore $r_x < r$.

The set of three curves labelled SIC are social indifference curves. These curves represent society's preference over combination of current and future consumption. These social indifference curve slope downward. The slope of the SIC_2 at X is –(1+p), where p_x is society's marginal rate of time preference or the marginal social rate of time preference at X.

Figure 2 – Equity of the MRTP and the interest rate



The optimal point is at X in Figure 2.

At X, the slope of the social indifference curve, -(1 + p_x) with p_x is the marginal social rate of time preference, equals the slope of the consumption possibility frontier, -(1 + r_x) with r_x the market interest rate.

Consequently, the marginal social rate of time preference, p_x equals r_x, the marginal rate of return on investment.

Furthermore, at point X these rates would also equal the economy-wide market interest rate, i.

Finally, at X, all individuals have the same MRTP because if their MRTP > i they would borrow at i and consume more in the current period until their MRTP = i and if MRTP < i they would invest until their MRTP = i. Since everyone's MRTP equals i, it would be the unanimous choice for the social discount rate.

Real economy

An actual economy (with taxes, risk and transaction costs) would not operate at the optimal point X, but at a point such as Z.

Here, society would under invest and rx > px.

Furthermore, because different people face different tax rates, risk and costs, numerous values exist for both MRTPs and the marginal rate of return on investment.

Thus, there is no obvious choice for SDR, in practice.

2. Alternative SDR methods in the absence of perfect markets

This section discusses five potential discounting methods.

Three methods assume that the social discount weights decline at a constant rate but they differ in the rate of decline.

1. One of these methods uses a social discount rate equal to the marginal rate of return on private-sector investments, r_z;

2. a second uses a social discount rate equal to the marginal social rate of time preference, p_z;

3. a third uses a social discount rate equal to a weighted average of p_z, r_z, and i, where i is the government's real, long-term borrowing rate. The weights should reflect the amount of the project's resources that are financed by consumption, investment, and foreign borrowing, respectively.

4. A fourth method, which is called the shadow price of capital method, entails distinguishing between project impacts that affect investment and those that affect consumption. Changes in investment are weighted by a parameter, which is greater than one, called the shadow price of capital, denoted by θ. The resulting changes in "consumption equivalents" and the changes in consumption are then discounted at p_z.

5. A fifth method uses a discount rate that declines over the time horizon of the project.

6. A sixth method, which is discussed in Appendix 10B, discounts benefits and costs using s_G, a rate based on the growth in real per capita consumption. Each method generally obtains a different NPV for the same project.

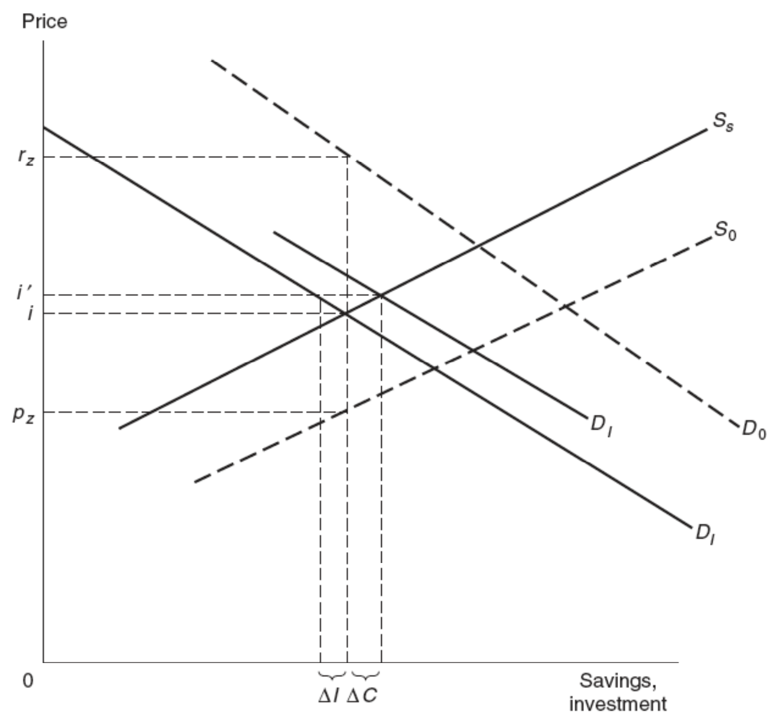
Using the Marginal Rate of Return on Private Investment (rz)

The argument for using the marginal rate of return on private investment as the social discount rate is that, before the government takes resources out of the private sector, it should be able to demonstrate that society will receive a greater rate of return in the public sector than it would have received had the resources remained in the private sector.

Therefore, the return on the government project should exceed r_z, the marginal return on private investment.

The most compelling case for the use of r_z was made by Arnold Harberger who analysed a closed domestic market for investment and savings, such as the one presented in Figure 3.

Figure 3. The optimal level of consumption and investment in a two periods model



. In the absence of taxes and government borrowing, the demand curve for investment funds by private-sector borrowers is represented by D_o and the supply curve of funds from lenders (or savers) is represented by S_o.

With corporate taxes and personal income taxes, the demand and supply curves would shift, resulting in a market clearing rate of *i* and a divergence between r_z and p_z, as discussed previously.

Harberger assumed that a government project would be financed entirely by borrowing in a closed domestic financial market. The demand for funds for the new project would shift the market demand curve to D_I', the market rate of interest would rise from i to i', private-sector investment would fall by ΔI and private-sector savings would increase by ΔC.

As the increase in private-sector savings exactly equals the decrease in private-sector consumption, the project would "crowd out" both investment (by ΔI) and consumption (by ΔC).

Harberger suggests that the social discount rate should be obtained by weighting r_z and p_z by the respective size of the relative contributions that investment and consumption would make toward funding the project. That is, he suggests that the social discount rate, s, should be computed as follows:

$$s = ar + (1 - p)p_z \quad (8)$$

where,

$$a = \frac{\Delta I}{(\Delta I + \Delta C)} \quad (9)$$

and,

$$(1 - a) = \frac{\Delta C}{(\Delta I + \Delta C)} \quad (10)$$

Finally, Harberger asserts that savings are not very responsive to changes in interest rates.

This assertion, which is strongly supported empirically, implies that the S_S curve is close to vertical and, as a consequence, ΔC is close to zero. This, in turn, suggests that the value of the parameter a is close to one and the value of (1 - a) is close to zero.

In other words, almost all of the resources for public-sector investment are obtained by crowding out private-sector investment. Thus, Harberger suggests that the marginal rate of return on investment, r_Z, is a good approximation of the true social discount rate.

Numerical Values of r_z. The value of r_z is perhaps best thought of as the rate of return on low risk, private sector investment before taxes but after correcting for inflation. Starting with the nominal return on long term bonds and adjusting for taxes and inflation suggests that r_x is approximately equal to 0.08 (i.e. the SDR = 8%, with sensitivity analysis + or - 2%).

Criticisms of the Calculation and use of r_z. There are several criticisms of both the use of r_z and of its estimation. These criticisms suggest that using an SDR of 8 percent is generally an upper limit.

1. Private sector rates of return incorporate a risk premium. Therefore, if benefits and costs are measured in “certainty equivalents”, then using private sector rates would be “double counting”, i.e. it would account for risk in two ways.

2. A project might be financed by taxes rather than loans – hence, consumption would also be crowded out.

3. Some loans may be obtained from foreigners at a lower rate.

4. Private sector returns may be pushed upward by distortions caused by negative externalities and monopolistic pricing.

5. There is no fixed pool of investment where government investment replaces private dollar for dollar.

Thus, using r_z (and a SDR of 8%) should be viewed as upper limit for the SDR.

Using the Marginal Social Rate of Time Preference Method (p_z)

Many analysts hold that the SDR should be thought of as the rate at which individuals in society are willing to postpone a small amount of current consumption in exchange for additional future consumption (and vice versa).

In principle, p_z represents this rate. Consequently, many believe that the SDR should equal p_z.

Note also that if a government project is financed entirely by domestic taxes and if taxes reduce consumption, but not investment, it is appropriate to set a = 0 in equation (8), yielding an SDR equal to p_z .*Numerical Values of* p_z. In practice, the best return that many people can earn in exchange for postponing consumption is the real after-tax return on savings.

Therefore, one option is to use this rate as an estimate of p_z. Starting with the nominal, pre-tax interest rate on government bonds and adjusting for taxes on savings and inflation yields estimates of p_z between 0.00 and 0.04.

Thus, this method suggests using a real SDR = 2% with sensitivity analysis at 0% and 4%.

Criticisms of the Calculation and Use of $p_{z.}$ There are several criticisms:

1. Individuals differ in preferences and opportunities – some save and some borrow and some save by reducing debt. Since reducing some debt isn't taxed, people who do this earn a much higher after-tax return than other people. It is not clear how one can aggregate these different individual rates into a single SRTP?

2. Since individuals simultaneously pay mortgages, buy

government bonds and stocks and borrow on credit cards at high interest rates, it is unclear whether an individual has a single MRTP.

3. It does not take into account future generations.

Discounting Using the Weighted Social Opportunity Cost of Capital (WSOC)

If *a* is the proportion of the project's resources that displace private domestic investment, *b* is the proportion of the resources that are financed by borrowing from foreigners, and 1-*a*-*b* is the proportion of the resources that displace domestic consumption, then this method computes the social discount rate as the weighted average of these rates, called the weighted social opportunity cost of capital (WSOC):

$$WSOC = ar + bi + (1 - a - b)p_z \qquad (11)$$

where *i* is the government's real, long term borrowing rate. As $p_z < i < r_z$, it follows that $p_z < WSOC < r_z$. Obviously, the previous methods are special cases of this more general approach.

Numerical Values of WSOC. To compute WSOC, we suggest using r_x = .08, p_x = .02, and *i* is the 10-year US Treasury Bond adjusted for inflation (CPI) = .04. Estimates of *a* and *b* vary considerably. Different parameter values suggest 5% < WSOC < 7%, or approximately, 6 percent.

If the project is financed by taxes, then a crude approximation for *a* is given by the ratio of gross fixed investment to GNP, which equals 16.8%. Under these circumstances, WSOC =3 percent.

Criticisms of the Calculation and Use of WSOC

1. Criticisms about the calculation of p_z and r_z also apply to this method.

2. WSOC requires each project be evaluated by its own discount rate, which depends on the sources of its resources.

This is difficult to do and labour intensive.

The Shadow Price of Capital (SPC) Method

If all the resources used in a project displace current consumption, and all the benefits produce additions to future consumption, then the social discount rate should reflect social choices in trading present consumption for future consumption and p_Z would be the natural choice for the discount rate.

However, projects could produce costs and benefits in the form of consumption *or* investment.

Due to market distortions, the rate at which individuals are willing to trade present for future consumption, p_z, differs from the rate of return on private investment, r_z, as previously discussed. Thus, flows of investment should be treated differently from flows of consumption.

The shadow price of capital method converts investment gains or losses into consumption equivalents. These consumption equivalents, like consumption flows themselves, are then discounted at p_z.

The shadow price of capital method requires that discounting be done in four steps:

1. Costs and benefits in each period are divided into those that affect consumption and those that affect investment. (When in doubt use 15% investment, 85% consumption.)

2. Flows into and out of investment are multiplied by the SPC to convert them into consumption equivalents.

3. Changes in consumption are added to changes in consumption equivalents.

4. Resulting amounts are discounted at $p_{z.}$

A general expression for the shadow price of capital (SPC) is:

$$\theta = \frac{(r_z + \delta)(1\ \ f)}{p_z\ \ r_z f + \delta(1\ \ f)}$$

(12)

where r_z is the net return on capital after depreciation, δ is the depreciation rate of the capital invested, f is the fraction of the gross return that is reinvested, and p_z is the marginal social rate of time preference.

Numerical Values of the SPC. The SPC requires many parameter estimates; see equation (12). Using "reasonable" values of these parameters provides an estimate of SPC = 1.65. While there is considerable uncertainty about this parameter value, we suggest that $1.5 < SPC < 2.5$.

Criticisms of the Calculation and Use of the SPC. Although this method is theoretically appropriate:

1. It is difficult to explain how and why NPV calculations are made.

2. The method has relatively heavy information requirements.

3. The allocation of costs and benefits to investment and consumption is fairly subjective and open to manipulation

4. As q depends on p_z and r_z, the SPC suffers from the criticisms that apply to these parameters.

Time Declining Discount Rate

So far we've discussed only constant (time-invariant) SDRs. There are three reasons, however, to suggest the use of a time-declining SDR instead:
1. Empirical evidence suggests that people use lower discount rates for events that occur farther into the future. 2. Long-term environmental and health consequences have small PV when discounted using a constant rate.

3. Constant rates do not appropriately take into account the preferences of future generations.

A declining rate could be produced through proportional discounting or a sliding scale. Analysts are not in agreement on the use of declining social discount rates.

Conclusions on Social Discounting Methods in the Absence of Perfect Markets

The SPC method is theoretically preferable, but has computational disadvantages. If, for some reason, it is not possible to use the shadow price of capital method, we suggest using the weighted average social discount rate method using a discount rate of 3 percent with sensitivity analysis at 1 and 5 percent if the project is financed by taxes, or using a discount rate of 6 percent with sensitivity analysis at 5 and 7 percent if not.

For projects with large future (i.e. intergenerational) environmental or health effects, there is some justification for applying lower discount rates, especially to impacts that occur far in the future.

The discounting method based on the real growth rate suggests a discount rate of 3.25 percent, with sensitivity analysis at 2.1 percent and 5.25 percent.

Advocates of declining discount rate methods would probably suggest applying a slightly higher rate to impacts that occur within the next few years and would certainly suggest applying a lower rate to impacts that occur far in the future. One should always perform sensitivity analysis.

IV. FURTHER TOPICS

Adjusting the social discount for systematic risk

This sub-section focuses on whether and how the SDR for a project should be adjusted for the risk of the project when the analyst does not compute benefits and costs in terms of certainty equivalent values.

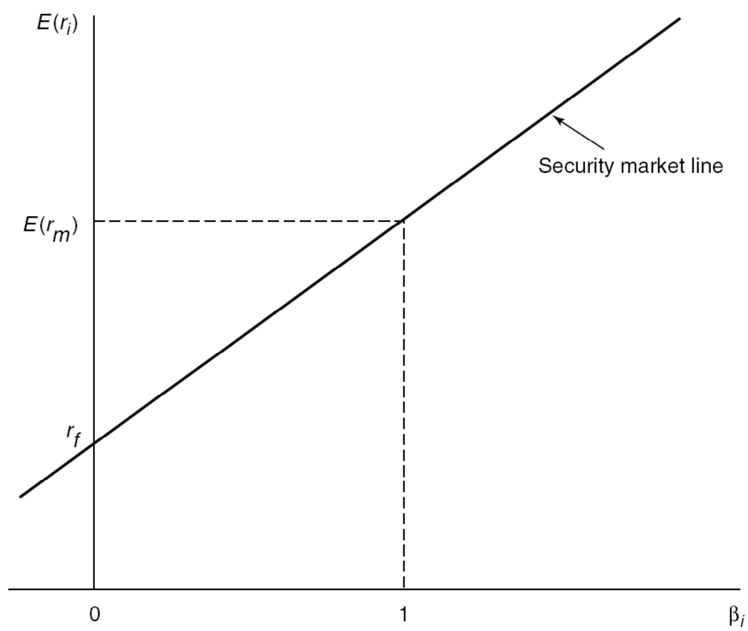
There are two types of risk: unique risk (or unsystematic risk), which pertains to a particular investment, and systematic risk, which pertains to the effects of economy-wide changes. Unsystematic risk can be reduced by diversification; systematic risk cannotThe capital asset pricing model (CAPM) provides guidance about how to take account of systematic risk. It states that that, in equilibrium, the expected rate of return on an asset equals the risk-free rate plus an amount that compensates for systematic risk:

$$E(r_i) = r_f + \left[E(r_m) - r_f\right]\beta_i \tag{12}$$

where $E(r_i)$ is the expected rate of return on individual investment i, $E(r_m)$ is the expected rate of return to the market portfolio, r_f is the risk-free rate of return or the rate of return on a risk-free asset, such as high-grade corporate bonds or T-bills, $\left[E(r_m) - r_f\right]$ is the market risk premium or equity premium, and β_i is a measure of systematic risk for individual asset i.

Often the systematic risk for a security is called its beta.

Figure 4. The capital asset pricing model



On the graph, in equilibrium all investment lies on the security market line with the expected return of an investment varying in direct proportion to its systematic risk, B_i.

It can be shown that :

$$\beta_i = \frac{\text{cov}(r_i, r_m)}{\sigma_m^2} \tag{13}$$

Thus a security's systematic risk-its beta- equals the covariance of the security's own rate of return with the market portfolio divided by the variance in the market rate of return. Setting $r_i = r_m$ and substituting (13) in (12) indicates that the market portfolio has a beta of one. As an investment that is uncorrelated with the market portfolio has a beta of zero and has shown by equation (12) has an expected return equal to r_f, the free risk rate of return.

Private-Sector Capital Budgeting Duplications of the CAPM

The private-sector capital-budgeting implications of the CAPM are straightforward.

First, estimate the beta of a project.

Second, use the right hand side of equation (12) to obtain the desired risk-adjusted rate of return. (This requires an estimate of the market risk premium, which is about 6 percent).

Third, discount the cash flows of the project using this risk-adjusted discount rate to obtain the project's NPV. As usual, proceed if the NPV > 0.

In practice, especially for new projects, beta is not known and is estimated by using rules of thumb:

1. Cyclical investments have beta > 1.

2. Investments highly correlated with the market portfolio (e.g. SP 500) have beta = 1.

3. Investments weakly positively correlated with the market have 0 < beta < 1.

4. Investments uncorrelated with the market have beta = 0.

5. Countercyclical investments have a negative beta.

Use of the CAPM in CBA

Rule 1. "Government projects with expected net benefits that are negatively (positively) correlated with national income should be discounted at a rate lower (higher) than the risk-free rate".

Rule 2. "*However, some argue that adjusting discount rates for risk is probably only worthwhile in practice for projects with clearly negative correlations and long horizons time*".

Social discount rate based on the real growth rate

Many economists have argued that current, market-based interest rate methods do not take future generations into account appropriately[1].

They argue that society should treat all generations' welfare equally but should consider that future generations will likely have higher per capita consumption than the current generation due to ongoing economic growth.

---

[1] Franck Ramsey « A mathematical Theory of Saving » ; Stephen Marglin « The Social Rate of Discount Rate and the Optimal Rate of Investment » *Quartely Journal of Economics*, 77, n°1 1963, 995-111 and Kenneth Arrow « Intergenerational Equity and the rate of Discount in Long Term Social Investment », 1995, unpublished paper given to the IEA world congress.

Consequently, assuming that consumption has declining marginal utility, consumption by a future generation should have a lower weight than consumption by a present generation, where the rate at which the weights decline over time is proportional to the growth rate of per capita consumption – the higher the growth rate, the higher the SDR.

Specifically, some economists argue that the SDR, denoted as S_G for this method, should equal to the long-run rate of growth in per capita consumption, g, multiplied by a non-negative constant, e, which is an elasticity that measures how fast the social marginal utility of consumption falls as per capita consumption rises:

$$S_g = ge \qquad (14)$$

The parameter $e(0 \le e \le \infty)$ e (represents a social evaluation of the intergenerational distribution of income. It summarizes the key value judgment about how quickly the marginal utility of consumption declines as average consumption rises.

When $e = 1$, the relative weight on each generation's consumption equals the inverse of its relative consumption per capita. That implies that a 10% reduction in the consumption of the current generation, for example, from 20.000€ to 18.000€, is an acceptable trade off for an 10% increase of the consumption of a richer future generation, for example from 40.000€ to 44.000€. In other words, society weight poorer, current generation’s loss of each € of consumption as twice as important as a gain of 1€ to the richer, future generation. Because the richer generation (initially) has twice the level of per capita consumption.

Setting $e = 0$ would imply that there should be no discounting.

If society wishes to give more weight to the current generation, it can add a pure rate of time preference, d, to the product of g and e:

$$S_g = ge + d \qquad (14)$$

where d represents the rate at which society discounts future generations' welfare, even if all generations have equal consumption per capita (i.e., $g = 0$).

We estimate that real per capita consumption grew at 2.25 percent between1947-1998 in the US. With this estimate of g, and setting $e = d = 1$ percent, gives $s_G = 3.25$ percent. Sensitivity analysis with e ranging between 0.5 and 2 implies s_G ranges from 2.1 percent to 5.25 percent.

IV. THE SOCIAL DISCOUNT RATE IN PRACTICE

Discounting practices in government vary enormously.

Often the discount rate is prescribed by a government agency (e.g., Office of Management and Budget, Congress Budget Office). In North America, rates were as high as 10% but have recently trended lower. OMB now uses 8%. CBO uses lower rates; it favors the MRTP method and uses 2% +/- 2% (based on the U.S. Treasury borrowing rate). The General Accounting Office favors the average nominal yield on Treasury debt (maturing between one year and the life of project) less inflation.

The SPC approach is best but hard to use.

VI – EXERCISES

Question 1. Quinze ans auparavant, vous étiez le gouverneur du Massachusetts. Vous deviez décider s'il fallait supporter un projet de construction d'un pont et d'une autoroute surnommé « Big Dig ». La durée nécessaire à la construction de ce projet est estimée à 7 ans.

Le coût des matériaux de construction qu'implique ce projet est de 45 million $ par an. Celui de la main d'œuvre est estimé à 20 million $ par an.

Par ailleurs, le projet implique une interruption de la circulation intra urbaine durant toute la construction. Cette interruption accroît de 30 heures par an le temps de transport de 100 000 travailleurs. Tous les travailleurs sont rémunérés 15$ de l'heure (on fait l'hypothèse qu'il n'existe aucunes distorsions, et que le salaire reflète la valeur qu'un travailleur attribue à son loisir).

Le projet « Big Dig » a pour objet de faciliter la circulation intra urbaine. Il réduira de 35 heures par an le temps de transport des travailleurs, en comparaison avec celui pré existait avant le lancement du projet.

De plus, une partie de ce projet implique la destruction d'une autoroute surélevée et son remplacement par un parc public. L'état du Massachussetts a estimé que chaque travailleur valorisait le parc à 40$ par an. On fait l'hypothèse que seuls les travailleurs utiliseront ce parc. On suppose également que le gouvernement bénéficie d'un taux d'escompte de 5%, et que les travailleurs sont immortels.

Le projet débute en année 0, il dégagera des bénéfices au début de l'année 7 (c'est-à-dire que le projet s'étale sur 7 ans, de t = 0 à t = 6).

En tant que gouverneur, devez vous approuver la construction de ce projet ? Utilisez l'analyse coût bénéfice.

Après avoir effectué l'ensemble des calculs en question a), vous réalisez que les coûts estimés peuvent être incertains. Le coût des matériaux de constructions est estimé à 45 million $ avec 50% d'incertitude et à 100 millions $ avec 50% d'incertitude. En supposant que l'aversion au risque du gouvernement est nulle, le projet doit-il être réalisé ?