CHAPTER II – THE CBA's MODEL

I – INTRODUCTION

In this section, we give an outline of the **main ingredients** that make up the particular cost benefit model.

Each ingredient will be examined later in a separate chapter.

The basic idea (Marglin, 1968) is to **assess a project** (a policy) **comparing its cost and benefits**.

As the general model points out, the aim is to maximise the difference between B (Benefits) and C (Costs), then:

$$NSB = B-C \qquad (1)$$

(NSB=Net Social Benefit from a government policy)

The difference is **efficiency** of the project. It can be regarded as **the additional resources that are now available**.

The Net Social Benefit has to be discounted in order to add flows which occurs are different periods.

It then called the Net present value. The NPV is the appropriate decision rule to assess a project.

The greater the difference, the greater the contribution of the project.

When **no constraints other** than production possibilities exists, **all projects with a positive difference** must be approved

Table 1 – Simple choice

	B	C
P1	100	60
P2	60	100

When **only one project can be accepted** one chooses the project with the **highest benefit.**

II – BENEFITS AND COSTS

Under most circumstances **the changes in producers surplus, consumer surplus and government revenue provide a measure** of the monetary value of a government policy benefit and cost.

Because these value are in monetary terms (1) can be summarised:

$$NBS = \Delta CS + \Delta PS + \Delta \text{Gov Revenue} \qquad (2)$$

(NBS=Net Social Benefit from a government policy)

Price are an accurate measure of benefits and costs when the market is well functioning

Competitive markets tend to be a good estimate of benefits and costs while observed prices in **a distorted market tend to be a poor measure**.

2.1 – Costs

The **opportunity cost is the best measure for cost**

It is equal to the **benefit of the next best alternative**. Renouncing to this next best alternative is the condition to choose the current solution.

Burning a 500€ banknote has an opportunity cost of 505€ to in consideration that putting this amount money in a bank would increase the utility by the interest rate.

In a CBA assessment **accountable cost** must be used only if they **truly reflects the opportunity cost**. If not, they must be de biased instead of reflecting the true social cost of a resource.

If for example, the national Post office doesn't includes in it costs the cost of using it's building because it owns it from a very long time, analyst should include an amortization.

2.2 – Benefit and consumer's surplus

"*The utility for an individual of the consumption of a good or a service, such as it is estimated by the consumers, is at least equal to the price which they pay for this consumption*".

Actually, it **is more important**, since a certain number of consumers would be ready to pay more than what they pay indeed. It **is this difference which measurement the surplus of consumer**. This surplus thus indicates the benefit that the individuals draw from their consumption.

Measurement techniques

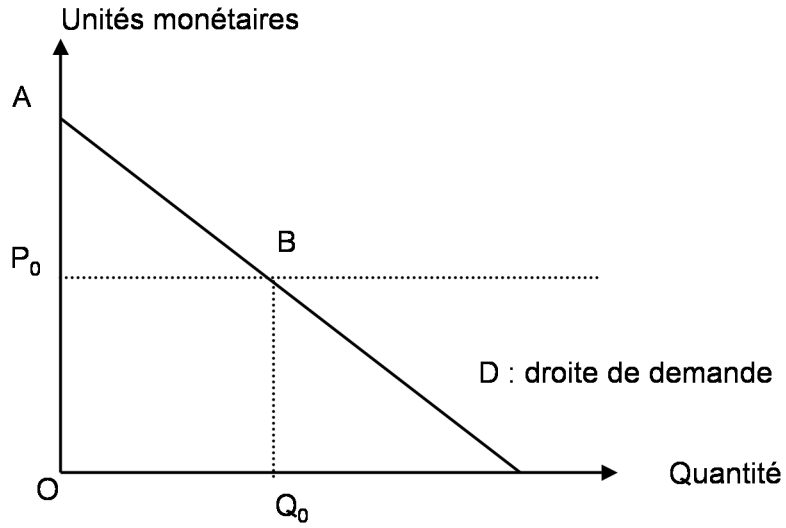
Le surplus du consommateur mesure la différence entre la valeur que les individus accordent à leur consommation de tabac et le prix que cette dernière leur coûte. Le prix est connu, mais le premier terme du calcul n'est pas observable. Toute la **difficulté réside donc dans le choix d'une technique permettant de calculer une valeur qui n'est pas directement mesurable sur le marché.**

Techniquement, on considère généralement que le surplus du consommateur est égal à la différence entre la disposition mar-

marginale à payer indiquée par la droite de demande et le prix effectivement payé.

$$SC(Q) = \int_0^{Q_O} p(Q)d(Q) - P_O.Q_O$$

Graphique 1 – Le surplus du consommateur



Si le prix du bien s'établit au niveau P0, les consommateurs demanderont la quantité Q0. Leur dépense pour le bien s'élèvera à P0Q0, représentée par la surface OP_0BQ_0. Or, la valeur accordée à chacune des unités consommées – indiquée par les points qui forment le segment AB de la droite de demande D - est supérieure au prix payé (sauf celle de la dernière unité consommée pour laquelle la disposition à payer est juste égale au prix) Les consommateurs bénéficient donc d'un surplus, mesuré par l'aire P_0AB.

Formellement

La fonction de demande, supposée ici linéaire, est de la forme

$$P = A - \beta Q \qquad (1)$$

Dans ces conditions, on montre que le surplus du consommateur Sc s'écrit :

$$Sc = 0,5 * P * Q * \frac{1}{e} \qquad (2)$$

Démonstration :

On sait que Sc = PAB. Or, la surface d'un triangle est égale à la moitié du produit de la base par la hauteur, i.e. S = (0,5 x AP x PB), soit S = (0,5 x AP x OQ) puisque PB = OQ.

Comme M = (OP x OQ), on peut écrire que OQ = M/OP, ce qui implique que :

$$S = (0,5 \text{ x } M \text{ x } AP/OP). \qquad (3)$$

Le rapport AP/OP doit être mis en relation avec l'élasticité e de la droite D, en négligeant le signe.

Une diminution de 100% de la quantité (de Q vers O) est associée à une augmentation des prix donné par le rapport AP/OP.

On sait par définition de l'élasticité que $\Delta Q = e * \Delta P$

Donc :

$$e * \frac{AP}{OP} = 100\% = 1$$

$$\Rightarrow \frac{1}{e} = \frac{AP}{OP}$$

On remplace alors $\frac{AP}{OP}$ dans (3) par sa valeur $\frac{1}{e}$ il vient (2).

Trois remarques :

1. Comme l'indique l'équation 2, **le surplus du consommateur varie en raison inverse de la valeur absolue de l'élasticité-prix**.

2. Lorsque **l'élasticité-prix est infinie**, c'est-à-dire que les consommateurs sont très sensibles au prix**, alors le surplus du consommateur est nul**. Dans le cas où l'élasticité est inférieure à 0,5 en valeur absolue, le surplus du consommateur est supérieur à la dépense.

3. Attention, il est probable que l'hypothèse de linéarité de la demande **conduit à sous-estimer le surplus du consommateur.**

2.3 – Benefit and producer's surplus

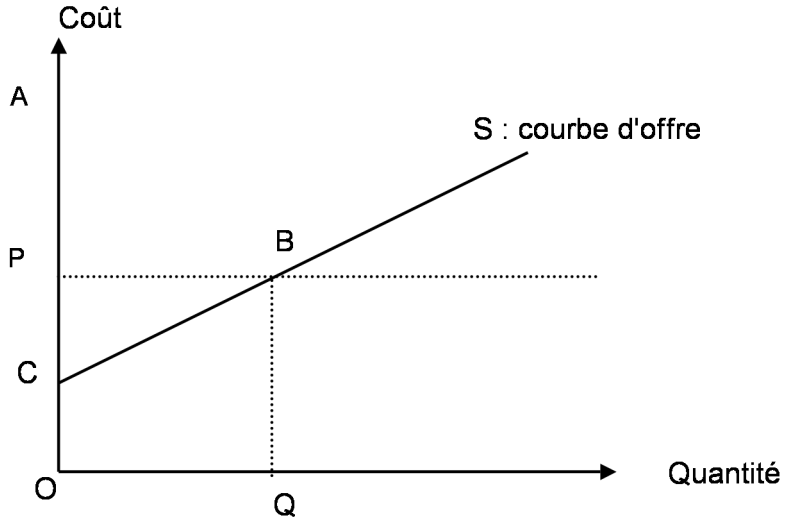
Afin de **mesurer l'impact complet de la production d'un bien sur la société, il convient de prendre également en compte le bénéfice dégagé par la production et la distribution du bien.** Comment mesurer ce bénéfice et quel est sa signification ?

Techniques de mesure

Le **bénéfice** engendré par la production est *« mesuré par la valeur de la production alternative qui pourrait être produite avec les ressources en capital et en travail qui sont mobilisées par la production de tabac (on raisonne ici en termes de coût d'opportunité) ».*

Le **surplus du producteur** est égal à la différence entre le revenu tiré de la production et le coût d'opportunité des ressources engagées. Il correspond à une **« rente »** constituée par les **sommes versées au-dessus du montant minimum nécessaire pour garantir que le bien sera produit**.

Dit autrement, **le surplus du producteur est égal à la somme qui pourrait être retirée aux producteurs sans venir diminuer la quantité produite.**



Graphique 2 – Le surplus du producteur

$$SP(Q) = PQ - \int_0^Q CM(Q)dQ \qquad (4)$$

$$SP(Q) = PQ - CV(Q) \qquad (5)$$

En effet :

$$CT(Q) = CV(Q) + CF \Rightarrow \frac{dCT(Q)}{dQ} = \frac{dCV(Q)}{dQ} = Cm(Q)$$

Or $\Pi(Q) = PQ - CT(Q)$

$\Rightarrow SP(Q) = \Pi(Q) + CF$

Pour accepter de produire une quantité quelconques, les producteurs doivent recevoir en échange, pour chaque unité, un montant au moins égal à celui qui est indiqué par la droite d'offre S. Or, toutes les unités produites seront en fait vendues au même prix.

Avec un prix P_0, les producteurs offriront la quantité Q_0 et dégageront alors un surplus égal à la différence entre le revenu tiré de la vente (représenté par la surface OP_0BQ_0) et la surface située sous la droite de demande ($OCBQ_0$), soit P_0BC.

Formellement

$$W_p = \frac{P * Q}{(1+\eta)} \qquad (6)$$

Trois remarques.

1. L'ampleur du surplus du producteur **dépend de l'élasticité de l'offre**. Si les quantités offertes sont très sensibles au prix (par exemple parce que les ressources engagées dans la **production sont facilement redéployables** vers une autre production), une petite baisse du prix se soldera par une réduction importante de la production. Cela se traduit **graphiquement par une droite d'offre relativement plate**.

Le surplus du producteur est alors d'autant plus faible que l'offre est élastique. Ainsi s'il est facile de trouver une produc-

tion qui vient remplacer de manière rentable la production du bien, une petite baisse du prix conduira à un changement important dans la production.

Les producteurs seront nombreux à préférer produir**e la production alternative.** Dans ce cas, l'offre du bien est relativement élastique. Le **surplus du producteur** sera d'autant plus petit que **l'offre est élastique**, ce qui se représente par une droite d'offre (S) plate, réduisant la surface du triangle PBC.

2. Lorsqu'il n'existe **pas de production alternative** suffisamment rentable, alors la **droite d'offre est inélastique** (verticale) et le **surplus du producteur est important**.

3. Notons enfin que la surface CBQ0O représente **le coût variable** total de production sauf si la production est monopolistique. Dans ce dernier cas, cette surface mesure le coût total.

III – ALTERNATIVE MEASURES OF CONSUMERS SURPLUS

Consider one individual faced by a single price change.

3.1 – Marshallian measure versus Hick's

Marshallian measure

The first case to consider is one where the price rise **is too large as to cause** the individual **to cease consuming** the product entirely.

There is a current level of satisfaction with the production and a level of satisfaction without the product. The difference is the Marshallian measure. Marshall (1942) defined consumer surplus as " *the excess of the price which he would be willing to pay rather than go without the thing, over that which he actually pay"*.

The compensating variation

When one refrains **the all or nothing comparison**, other measure of consumer surplus can be considered? These other measures are due to Hicks (1943).

The compensating variation (CV) is "*the amount of compensation that one can take away from individuals and leave them just as well off as before the change*".

Again, the change we are considering is a price reduction caused by an increase in the output from a public project.

The CV works under the assumption that the price change will occur. For this reason it is call a *"forward teste that is allowing the change to take place and trying to value the new situation. It asks what is the individual 's WTP for that change such as the utility level is the same before the price change took place*."

Although the concept is **forward looking**, the utility level after the WTP amount has been extracted from the individual to the original utility level.

The CV is also a WTP concept but it does not operate with the standard (Marshallian demand curve). As always, one is changing price, holding income constant. But, the price change has **an income effect** (the mower price means that the purchasing power has increased, and so moire can be spent on all goods) as well as a substitution effects (the lower prices for the public project means that other goods are relatively more expensive and their consumption will be reduced.

The CV tries to **isolate the substitution effect** and **eliminate the income effect**. It tries to establish how much more the individual is willing to purchase of the public project assuming that the purchasing power effect can be negated. The resulting price and quantity relation; with the income effect excluded I, is

the compensated demand curve. The area under the compensated demand curve measure the **CV of the price change**.

The equilibrating variation.

There is **as second way** of **isolating the income effect** which is called by Hicks the equilibrating variation (EV). This is defined as follows" *The amount of compensation that has to be given in order that an individual forgo the change yet be as well off as after the change*".

For the EV the price change **does not take place**. It is therefore called the **"backward test**". That is the individual is asked to value the forgoing of the change. The individuals is to *receive a sum of money to be as well off as if the change has taken place*. It is, **nonetheless, also a WTP concept,** in the sense that it records what others have to be willing to pay to prevent the individual having the benefit of the change.

The difference is that the CV measures the maximum WTP of the individual, while the EV measures the minimum that must be received by the individual.

There is an equilibrated demand curve to parallel the compensated one. **The income effect involves giving the individual sum of money to compensate for the purchasing power effect that is not being allowed to occur**.

The **income effect is being neutralized** but at a higher level of purchasing power. In this **way all that remains is the relative price effect**, the substitution effect as before. The area under the equilibrated demand curve measures the EV of the price change.

3.2 – Diagram

All three measures will now be explained in term of next diagram. We consider two goods, X and Y? the change that will be analysed is a fall off the price of good X. This can be thought to be caused by a public project for example, say the government builds a hydro-electricity plant, which lowers the cost f electricity ti consumers. Good Y, on the vertical axis, will be the numeraire (he unit I which relative values will be expressed).

The top half of the diagram presents the **consumer's indifference map** for X and Y, together with the budget constraint. An indifference curve shows all combination of the two goods that gives the individuals a particular level of utility. Curves to the north east show higher levels of satisfaction.

The budget line shows all combination of the two goods that can be purchased with a fixed income and a given set of consumer price for X and Y. The individual's aim is to reach the highest indifference curve, subject remaining on the budget line.

For the **specified budget line** Y_1X_1, the individual choose point A, which produces the level of satisfaction U_1 (A is the **tangency** point between the indifference curve and the budget line).

The **slope** if the budget line is determined by the ration of the prices $\frac{P_x}{P_y}$. Thus, when the price of X falls, the slope will flatten, causing the budget line to rotate outwards from Y_1. The new

budget line is denoted Y_1X_2. With the new relative prices, the individual choose point B on U_2.

The **bottom half** of the diagram **traces the implications** if the indifference curve analysis for the price ad quantity relation (i.e demand) for X.

The original ratio of relative prices defines the price P_1. At this price Q_1 is purchased, being the X co-ordinate f the A on the indifference curve diagram. The P_1 and Q_1 combination fixes the point a on the lower half of the diagram? In this way we move from point A on the indifference curve diagram to point a in the demand curve. With the lower ration of relatives prices, P_2 is defined. The individual by moving to point B on the top part of the diagram chooses Q_4 of the X. The P_2 and Q_4 pairing locates point b on the lower part. Connecting point a and b determines the traditional, **Marshallian demand curves**.

The CV and EV both **contain only substitution effects**. They represents movement along indifference curves.

For the CV**,** one is to be kept at the original level of satisfaction. The movement along indifference curve U_1 from point A to point D (D is where a budge line with the flatter slope is tangent to U_1). In termes of the diagram this translates to the point a and d. Connecting these two points forms the compensated demand curve.

Similarly the EV keeps the individual at the higher level of satisfaction U_2 and traces the substitution effect movement from point C to point B. (C is where a budget line with the ori-

ginal slope to tangent to U_2). The corresponding points on h lower point of the diagram are c and b. Connecting points c and b forms the equilibrated demand curve?

The consumer surplus effect of the price change P_1 to P_2 is given as the area under demand curve between these two prices.

Since **there are three demand curves**, we have **three separate measure.**

The Marshallian measure is the area P_2baP_1.

The CV is the area P_2daP_1

The EV is the area P_2bcP_1.

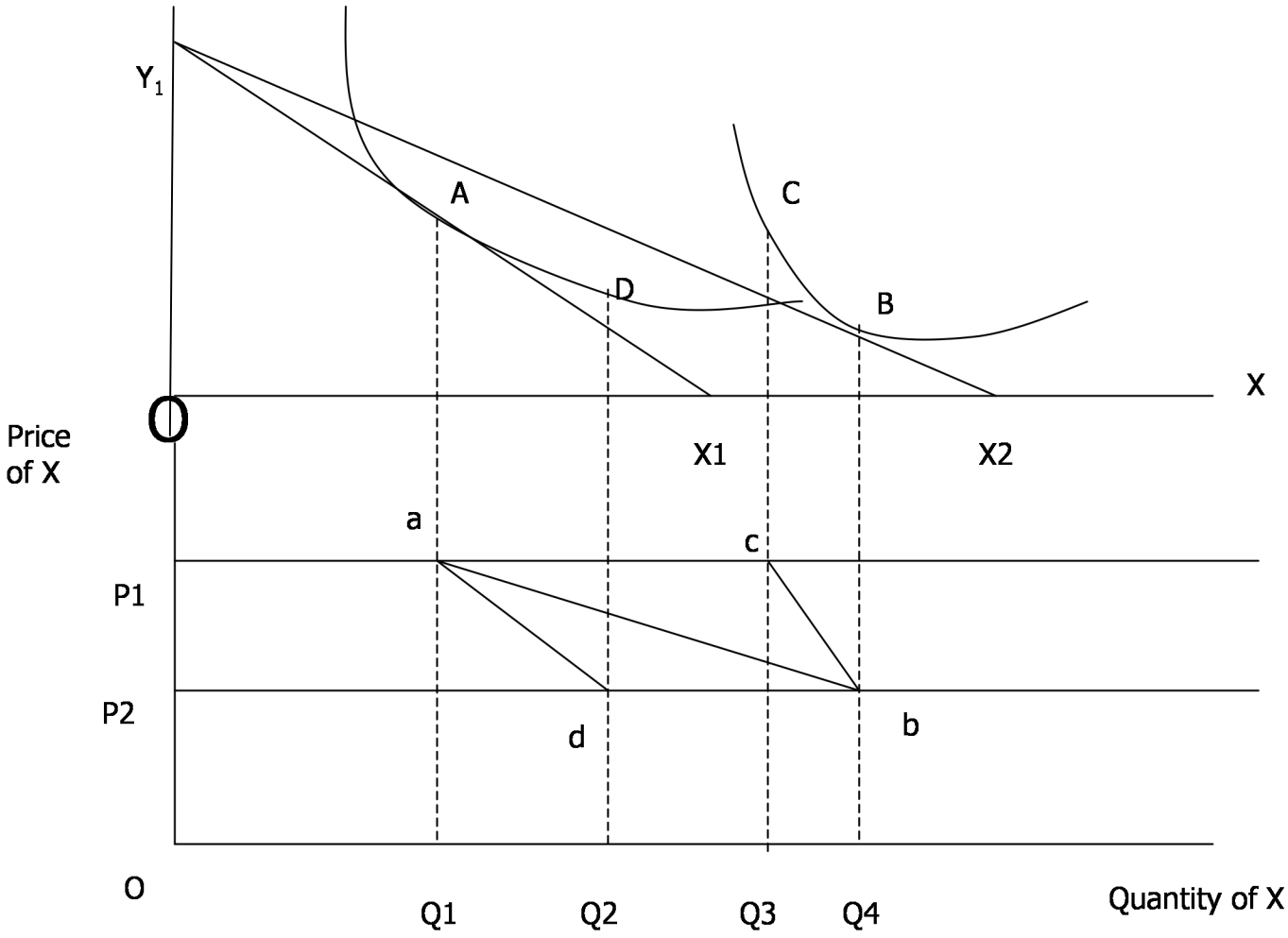


Fig 3 – Alternative measure to consumer's surplus

Marshallian measure: AB — var CS: P2baP1
Compensating variation: AD — var CS: P2daP1
Equilibrating variation: CB — var CS: P2bcP1

3.3 – Marginal utility of income

The relative sizes of the three measures are also shown in the next diagram.

The Marshallian measure is in between the smallest measure, the CV, and the largest measure, the EV.

CV<Marshallian<EV when price decrease

This ordering always holds for beneficial changes (where people are better off after the change than they were before the change) as with the price reduction we were considering. The order is reversed for adverse changes.

EV<Marshallian<CV when price increase

It is instructive to analyse further the relation between the CV and the EV.

The key to understanding the relative size of the measures lies in the concept of the **marginal utility of income**. It is usual to assume that the **marginal utility of income diminishes as income rise**. Thus, if one is at huger level of income, one will value higher in monetary terms because money income is worth less.

On the next diagram the marginal utility f income is on the vertical axis and income on horizontal axis. The curve relating the two variables from right to left. Consider a given sized utility changes, that is an area of a particular magnitude.

The income equivalent measured along the horizontal axis -is **larger with the level of income**. **Thus** even though the areas Y_1Y_2SR and Y_2Y_3TS indicate **equal sized** utility changes, the **income equivalent are different**.

The **higher income** reference point would value the change as Y_2Y_3, while the **lower-income** reference point would value the change as Y_1Y_2, a considerably **smaller amount**.

Fig 4. Marginal utility of income

We have just seen that the higher the utility, or real income , the higher one evaluates a good in monetary terms.

Thus for a beneficial change one's money evaluation **is greater after than before the change.** *Since the EV tries to make individuals as well off s they would have been with the change, it must involve a larger amount than the CV, which tires to make people as well as before the change occurred.*

Which to use depends on the purpose.

CV is preferred in theoretical work. But often the legal system decides who should compensate whom by the allocation of property rights.

For example, the resident near a proposed airport are to made as well off with the airport as they where previously? Builders of the airport must pay the residents to forgo their peace and quiet. Resident are not expected to have to pay the airport authority to refrain from building the airport.

IV – EXTERNALITIES

The presence of externality changes the CBA criterion:

$$NPV = \Delta SC + \Delta SP + \Delta GVR \pm X \qquad (7)$$

Buchanan and Stubblebine (1962) provided a **battery of definitions** of externality, which are very useful for public policy purpose.

They define **when an externality exists** and **where there is and there is not an externality problem.**

4.1 – When externality exists?

Externality **is said to exists** when there is interdependence between utility or production functions of individuals:

$$U^A = U^A(X_1, X_2, ..., X_m, Y_1) \qquad (8)$$

This states that the utility of individual A is dependent on the activities X_1, X_2,…, X_m that are under **his** control but also on an other Y_i which is **under the control of second person**, individual B.

The outside activity Y_1 can **enhance** A's utility, for example, if B is a gardener who grows beautiful flowers that decorate A's

neighbourhood) or can **detract** from A's utility (for example if B is a smoker who indirectly cause the non smoking A to get a cancer).

4.2 – When an externality is potentially relevant?

Two aspects of the above specification are important.

1. The marginal utility to A from Y_1 should not be zero. For example, I may not care whether a person smokes or not. In this case the smoking does not cause an externality to me.

2. If A is not affected when **B is in his best position**, then this would **not be an important external effect**. For example, I may care whether another person smokes. But if that person choose not to smoke, then again we do not have an externality.

These two considerations lead to a **more precise formulation**. The fact that he and me wear shoes is not the proof of interdependence between he and me.

Let's B's equilibrium value of Y_1 be denoted Y_1^*. Let's denote A's marginal utility MU from Y_1 by MUY^A_1.

A "**potential relevant externality"** *is when "the activity actually performed generates any desire on the part of the affected party A, to modify the behaviour of the part empowered to take action B, through trade, persuasion, compromise, convention. If I can't stop the activity of the other party, there is not externality*."

$$\text{As long as } MU^A_{Y1} \neq 0 \text{ (and } Y_1 = Y_{1*}) \qquad (9)$$

holds, an externality remains (**utility function are interdependent**).

It is called **potentially relevant** because A would like B's behaviour to adjust (produce more or less) and there is the potential for someone to gain.

An externality **is positive** as long as (3) is positive and **negative** if (3) is negative.

When (3)=0 the externality is irrelevant (for public purpose).

4.3 – When an externality is relevant?

The removal of an externality will promote losses as well as gains.

B will no longer be in his best position. So not all potentially relevant losses are necessary to be modified. That is, it may not be efficient to change the existing externality. **The mere existence of an externality does not necessarily imply inefficiency, and hence government intervention**. This leads to a **final refinement** in the definition of an externality.

A Pareto relevant externality is when: "*The extent of the activity may be modified in such way that the externality affected party A can be made better of without the acting party B being worse off*."

To formalize this definition we need some statement of what optimising behaviour B will be engaged in, in the absence of consideration about A.

Let's B marginal cost of engaging in Y_1, be denoted MC^B_{Y1}. In equilibrium any additional satisfaction will just equal the additional cost and hence:

$$MU^B_{Y1} = MC^B_{Y1} \qquad (10)$$

The **externality will be Pareto relevant** when the gain to A (from a change in the level of Y_1) is greater than the loss to B (who has to move from his equilibrium level Y_1, thus making the left side of (4) smaller than the right.

That is, a **Pareto-relevant externality** is where :

$$MU^{A}_{Y1} > \left[MC^{B}_{Y1} - MU^{B}_{Y1} \right] \qquad (11)$$

The **externality is irrelevant** when both sides of the expression (5) are equal.

Example.

Say there is a shoe factory that produces marginal profits (benefits) given by the MB curve. The factory causes smoke which lead to external costs to an adjacent laundry-given by the MC curve.

A profit maximising factory would produce up to the point where marginal profit are zero. Equilibrium for the factory would therefore be at Q_3. For any scale of output between 0 and Q_3, an externality exists (the laundry has the interdependence). Between 0 and Q_2, e.g. at Q_1, there is a Pareto relevant externality (the MB is greater than the MC). The social optimum is at Q_2, where the MB is equal to the MC. There is an externality at Q_2, but it is not Pareto relevant (it not possible to make the laundry better of without making the factory worse off).

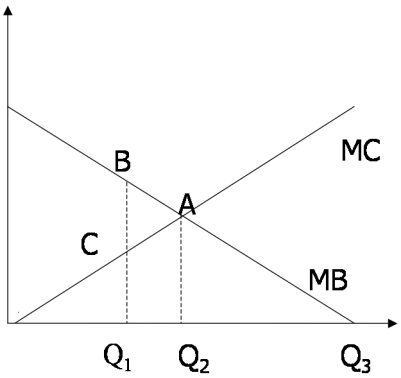


Figure 5. Laundry versus factory

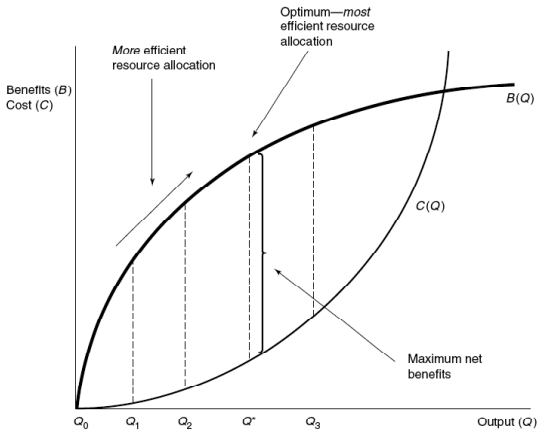
V – DECISION RULE

I already said ((l) that the Net Present Value is the appropriate decision rule.

In facts, there is some confusion about the appropriate decision rule. Both the Internal Rate of Return and the Benefit Cost Rtaio have been proposed as decision rules. The Appropriate criterion to use is the NPV rule. Other methods may give incorrect answer, the NPV rule des not.

An obvious caveat about NPV criterion is that it only applies to the actual alternatives specified. Other alternatives might conceivably be better. Although the criterion results *in a more efficient allocations of resources,* it *does not necessarily recommend the more efficient allocation of resources*.

Figure 6. More efficient allocation



Moving from Q_0 toward Q^* increases efficiency; that is: $NPV(Q^*) > NPV(Q_2) > NPV(Q_1) > NPV(Q_0)$

Moving beyond Q^* reduces efficiency, but Q_3 is more efficient than Q_0: $NPV(Q^*) > NPV(Q_3) > NPV(Q_0)$

Consider a project for which the alternatives vary along n output scale (Q). The benefit and costs associates with alternatives scales are represented by the function B(Q) and C(Q) respectively. The benefits increases s the scale increases but at a decreasing rate. In contrast, cost increase ant an increasing rate. A small project, for example Q_1 has a positive net benefits relative to status quo Q_O As the scale increases, the net benefits increases up to the optimal scales, Q_*. As the scale increases beyond Q_*, the net benefit decreases. Net benefit are positive as long as the benefit curve is above the cost curve, there are zero when the cost curve and benefit curve intersect and are negative for larger scale projects.

Suppose that the analyst evaluates only two alternatives output levels, Q_1 and $Q_{2,}$ relative to the status quo. Clearly Q_2 is preferred to output level Q_1, which in turn is preferred to the status quo. The analyst will therefore recommend Q_2. However the figure shows net social benefit are maximised at output Q_* thus optimal level output was not recommended because it was not among the set evaluated. In this example, use of the BPV leads to a more efficient alternative to the status quo but not to the most efficient alternative.

V – CONCLUSION

When observed **prices reflect the societal value of goods** (i.e when markets are efficient) **surplus variations are good to measure welfare variation induced by a public project.**

When observed **prices don't reflect the true societal value** of a good accurately (or when prices don't exists, i.e. Public Goods, externality), a process called **Shadow Pricing** is used.

Shadow Pricing is when observed prices are adjusted (or values are assignated when observed prices don't exist) so

that they come as close as possible **to measuring the social value of the good in question.**

QUESTIONS

Question 1. Dans quelles conditions le marché n'assure-t-il plus une allocation optimale ?

Question 2. Deux amis achètent des billets pour aller au concert des Rolling Stones, 100 euros le billet. Devant, l'entrée deux brésiliens les abordent. Ils sont venus de Rio pour le concert, ils ont perdu les billets. Ils sont prêts a racheter aux deux amis les billets pour la somme de 1000 euros chaque billet. Les deux amis se consultent et refusent. Il n'y a aucune « arnaque » possible (faux billets, violence,etc.). Quel est le coût pour chacun des deux amis d'avoir été au concert.

Question 3. Pourquoi se fonder sur l'analyse du surplus pour étudier l'efficacité d'une politique économique ?

Question 4. Variation sur la courbe de demande et mouvements de la courbe.

Question 5. Les surplus sont-ils de bons indicateurs du bien-être ?